Stable propagation of pulsed beams in Kerr focusing media with modulated dispersion

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We propose the modulation of dispersion to prevent collapse of planar pulsed beams that propagate in Kerr-type self-focusing optical media. As a result, we find a new type of two-dimensional spatiotemporal soliton stabilized by dispersion management. We have studied the existence and properties of these solitary waves both analytically and numerically. We show that the adequate choice of the modulation parameters optimizes the stabilization of the pulse.

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1. INTRODUCTION

The analysis of the propagation of high-power pulsed laser beams is among the most active fields of study in nonlinear optics, in which the main dynamical phenomenon is the dependence of the refractive index of the materials with the amplitude of light fields. For propagation in materials showing a linear dependence of the refractive index with the laser intensity (optical Kerr effect), the mathematical formulation of the beam dynamics is adequately described by the cubic nonlinear Schrödinger equation[1] (NLSE). In this case, the excitation of optical solitons is one of the most significant phenomena.[2–5]

Despite the success of the concept of the soliton, these structures mostly arise in one-plus-one-dimensional configurations. An important exception are quadratic materials, which display a linear dependence of the refractive index with the amplitude of the field. In this kind of media, multidimensional soltions were predicted theoretically[6,7] and verified experimentally.[8] However, in Kerr media this is not possible, mainly owing to the well-known collapse property of the cubic NLSE in multidimensional scenarios. This implies that a two-dimensional laser beam that propagates in a Kerr-type nonlinear medium will be strongly self-focused to a singularity if the power exceeds a threshold critical value, whereas for lower powers it will spread as it propagates.

In that collapse prevents the stability of multidimensional soliton bullets in systems ruled by the cubic NLSE, a great effort has been devoted to search for systems with stable solitary waves in multidimensional configurations.[9–11] It has been recently shown that a modulation of the nonlinearity along the propagation direction in the optical material can be used to prevent the collapse of two-dimensional laser beams.[12,13] The concept has been extended to the case of several incoherent optical beams[14] and to the case of matter waves.[15,16]

In this paper we use a similar idea to stabilize against collapse of pulsed laser beams propagating in planar waveguides. This configuration is of great importance, as it is the common experimental procedure used for exciting spatial optical solitons. Instead of making a modulation of the nonlinearity, our idea is to act on the chromatic dispersion term of the NLSE. Thus, our procedure resembles that used to obtain dispersion-managed temporal solitons in optical fibers. Preliminary results on this kind of system were described by Bergé *et al.*[17] and more recently by Matuszewski *et al.*[18] In this paper we present a exploration in a wide range of parameters showing that an adequate choice provides an optimum stabilization for long propagation distances.

2. PHYSICAL MODEL

We consider the paraxial propagation along z of a pulsed beam of finite size in time t and spatially confined by a waveguide along the y axis. Thus, diffraction acts only in the x direction and is balanced by the self-focusing nonlinearity given by a refractive index of the form $n=n_0+n_2|\mathbf{E}|^2$. The dynamics of the slowly varying amplitude of the pulse $\mathbf{E}$ is described by a one-plus-two-dimensional NLSE of the form

$$2i\left(\frac{\partial \mathbf{E}}{\partial z}+k_0'\frac{\partial \mathbf{E}}{\partial t}\right)+k_0''\frac{\partial^2 \mathbf{E}}{\partial t^2}+\frac{1}{k_0}\frac{\partial^2 \mathbf{E}}{\partial x^2}+2k_0\frac{n_2}{n_0}|\mathbf{E}|^2\mathbf{E}=\mathbf{0}, \tag{1}$$

where k_0, k_0', and k_0'' are, respectively, the wavenumber in vacuum, the inverse of the group velocity, and the group-velocity dispersion coefficient.

As we mentioned above, for continuous beams an adequate modulation of n_2 along z prevents collapse of the light distribution and yields to stabilized two-dimensional solitons. From a mathematical point of view, one can achieve an equivalent effect by suitably modifying the second-order derivatives of the propagation equation.[19–21] However, it is physically impossible to change the sign of diffraction of a continuous beam during its evolution. For the case of pulses, only the temporal derivative in Eq. (1) can be modulated by one's changing the dispersion parameter k_0'', which can take positive as well as negative values. In this paper we will show that it is possible to stabilize a pulsed beam against collapse by acting only on the dispersion during beam propagation, without altering diffraction.

For a linearly polarized Gaussian pulsed beam with power $P=\int|E|^2\mathrm{d}z\mathrm{d}x$, an initial beam waist $w_{0,x}$ and temporal width $w_{0,t}$, it is useful to write the above Eq. (1) in adimensional form by means of the Fresnel length $F=k_0w_{0,x}^2$ in the reduced-time frame. Thus, we make the rescaling: $\tau=(t-k_0'z)/w_{0,t}$, $\eta=z/F$, $\chi=x/(F/k_0)^{1/2}$, and $u=(w_{0,x}w_{0,t}/k_0')^{1/2}E/P^{1/2}$. Then, if we consider that dispersion is modulated by a periodic function along propagation, we will finally yield to a generalized NLSE of the following form:

$$i\frac{\partial u}{\partial \eta}+\frac{1}{2}d(\eta)\frac{\partial^2 u}{\partial \tau^2}+\frac{1}{2}\frac{\partial^2 u}{\partial \chi^2}+g|u|^2u=0. \tag{2}$$

Here, $d(\eta)=Fk_0''/w_{0,t}^2$ and $g=k_0n_2PFk_0'/n_0w_{0,x}w_{0,t}$. It is not obvious what kind of periodic function $d(\eta)$ will be more adequate to stabilize pulsed beams. A simple choice (and natural for applications) is to take $d(\eta)$ as a piecewise-constant function of the form $d(\eta)=d_a$ for $\eta\leq\eta_a$ and $d(\eta)=d_b$ for $\eta_a<\eta\leq\eta_b$ (see top of Fig. 1) where d_a, d_b, η_a, and η_b are free parameters that can be varied to fit experimental requirements and to optimize the stabilization of the pulsed beam.

For usual glasses with second-order dispersion $k_0''\approx 10\ \mathrm{fs^2\,mm^{-1}}$, the adimensional values of our analysis correspond to an experiment performed with a mode-locked laser with wavelength $\lambda_0=785$ nm, beam waist $w_{0,x}=0.2$ mm, and pulse length $w_{0,t}=20$ fs. This yields to values of the Fresnel length $F\approx 320$ mm, and thus $\eta_a=0.02$ corresponds to 6.4 mm length layers. All these quantities are common in usual experiments with commercial mode-locked lasers. If glasses with high nonlinearity are used,[22] such as Ga:La:S ($n_2\approx 2\times 10^{-4}\ \mathrm{cm^2/GW}$), and the previous data are taken into account, the peak power of the pulse for our simulations is in the range of 5 KW, which can be easily achieved experimentally with available femtosecond mode-locked lasers.

3. VARIATIONAL ANALYSIS

To get some insight on the dynamics given by Eq. (2), we have performed an analytic study by means of the time-dependent variational approach.[23] Notice that Eq. (2) can be obtained from the Lagrangian density:

$$2\mathcal{L}=i(u\dot{u}^*-u^*\dot{u})+d(\eta)\left|\frac{\partial u}{\partial \tau}\right|^2+\left|\frac{\partial u}{\partial \chi}\right|^2-g|u|^4, \tag{3}$$

where the overdot denotes the derivative with respect to η. We choose a Gaussian ansatz of the form

$$u=A\exp\left[-\left(\frac{\chi^2}{2w_\chi^2}+\frac{\tau^2}{2w_\tau^2}\right)+i(\beta_\chi\chi^2+\beta_\tau\tau^2)\right]. \tag{4}$$

The η-dependent parameters in the above equation have the following meaning: A is the amplitude; w_χ, w_τ are the spatial and temporal widths; and β_χ, β_τ are the initial curvature and chirp. Although Gaussians are not exact solutions of Eq. (2), our choice simplifies the calculations and is a usual light distribution in experiments. The standard variational calculations[23] lead to the equations describing the dynamics of the pulsed beam spatial and temporal widths:

$$\ddot{w}_\chi=\frac{1}{w_\chi^3}-\frac{1}{2\pi}\frac{g}{w_\chi^2w_\tau}, \tag{5a}$$

$$\ddot{w}_\tau-\frac{\dot{\mathrm{d}}(\eta)}{\mathrm{d}(\eta)}\dot{w}_\tau=\frac{d^2(\eta)}{w_\tau^3}-\frac{1}{2\pi}\frac{gd(\eta)}{w_\tau^2w_\chi}. \tag{5b}$$

Numerical simulations of Eqs. (5) show that the beam can be stabilized for many choices of the model parameters. These equations also predict two types of oscillation for width w_τ: a fast one due to the modulation of dispersion and a slow one that almost coincides with the variation of w_χ. This low-frequency oscillation is generated by the internal nonlinear dynamics of the system.

4. LAYERS OF EQUAL SIZE

Variational models usually provide only simple and intuitive predictions that must be complemented with the integration of Eq. (2). In the first place, we will present a detailed analysis of the simplest case: two layers of equal size and different dispersions. All the results to be presented in this paper are based on direct simulations of Eq.

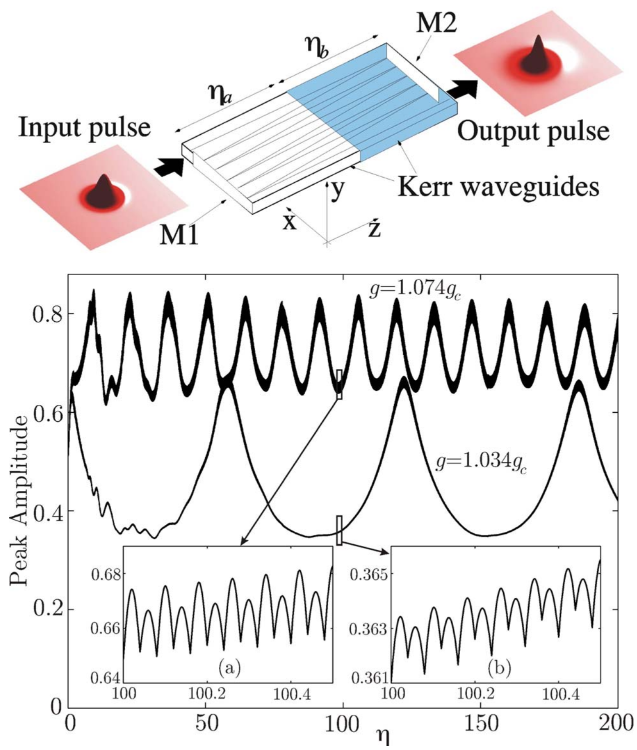

Fig. 1. (Color online) (top) Sketch of the propagation of a pulsed beam through a system with periodic modulation of dispersion made by our joining two different planar Kerr waveguides. The forward–backward trajectory can be obtained by reflection at mirrors M1 and M2. (bottom) Oscillations of the peak amplitude of stabilized pulsed beams propagating in the above system. The upper curve corresponds to a value $g=1.074g_c$ (surface plots in the top figure). For the lower curve, $g=1.034g_c$. The modulation is of the form $d_a=1+8$, $d_b=1-8$, $\eta_a=\eta_b=0.02$. In (a) and (b) details of the fast oscillations are shown.

(2) by using a split-step Fourier method on a 520×520 grid and absorbing boundary conditions to get rid of radiation.

In the first place, we must notice that, for constant anomalous dispersion and a given value of the beam flux P, there is a critical threshold for the nonlinearity g_{cr} for which a stationary solution of Eq. (2) exists. This is the well-known unstable Townes soliton, $u(\chi,\tau,\eta)=\Phi_\lambda(\chi,\tau)\exp(i\lambda\eta)$, where λ is a free scaling parameter. Owing to the scaling invariance of the cubic NLSE, a family of Townes solitons can be generated by making $\Phi_\lambda(r,t)=\lambda\Phi(\lambda\chi,\lambda\tau,\lambda^2\eta)$. Thus, the stretching symmetry of the cubic NLSE allows a continuum of Townes solitons with different sizes and different peak amplitudes. The condition for collapse is fixed by g_{cr} for a given value of P.

The size of the layers and the amount of dispersion can vary in a wide range. Taking into account previous studies on soliton stabilization by modulation of the nonlinearity,[14–16] we have centered our simulations around values of η_a and η_b in the range of 0.02, which corresponds to a layer of 6.4 mm thickness for a Fresnel length $F=320$ mm. The amount of dispersion varies asymmetrically around zero in the form of $d_a=1+8$, $d_b=1-8$. This choice minimizes radiation losses during propagation. It is important that the distribution of dispersion is asymmetric to keep the pulse below the collapse threshold. If the average dispersion is positive or zero, the beam will collapse. We must notice, however, that the range of the parameters for stabilization is wide. Self-confined propagation can be obtained at least within an order of magnitude of variation around the previous values. In addition, we will show below that stabilization is robust against addition of noise.

In Fig. 1 we plot the evolution of the peak amplitude of two different pulsed beams in a system sketched in Fig. 1 (top). In Fig. 1 (bottom) the upper curve corresponds to a value $g=1.074g_c$, where $g_c>0$ is the critical value of the nonlinearity for collapse. For the lower curve, $g=1.034g_c$. The modulation is of the form $d_a=1+8$, $d_b=1-8$, $\eta_a=\eta_b=0.02$. In Figs. 1(a) and 1(b), details of the fast oscillations are shown.

In Fig. 2 we plot the width oscillations of the stabilized soliton from Fig. 1 with $g=1.074g_c$. As the variational equations predict, the spatial width displays only a low-frequency oscillation, whereas the temporal width presents small-amplitude fast variations over the same low-frequency main oscillation. This is clearly seen in the frequency spectra displayed in Figs. 2(c) and 2(d).

To check the validity of the variational approach developed in Section 3, we compare the predictions of Eqs. (5) with the results of the numerical integration of Eq. (2) corresponding to the case shown in Fig. 2. That is, the modulation is of the form $d_a=1+8$, $d_b=1-8$, $2\eta_a=2\eta_b=0.04$. As can be seen in Fig. 3 there is a minor quantitative deviation in both the period and the amplitude of the oscillations. Nevertheless, the variational equations give a good qualitative behavior of the oscillations and, therefore, the range of parameters to be used in direct numerical simulation of the full NLSE.

In Fig. 4 we plot the results of the simulation of Eq. (2) for the same input pulsed beams of Fig. 1. In this case we have chosen waveguides of half size, resulting in a frequency of modulation that is twice that of Fig. 1. Stabilization of both beams is achieved, showing that there is a wide range of the values of the parameters involved. How-

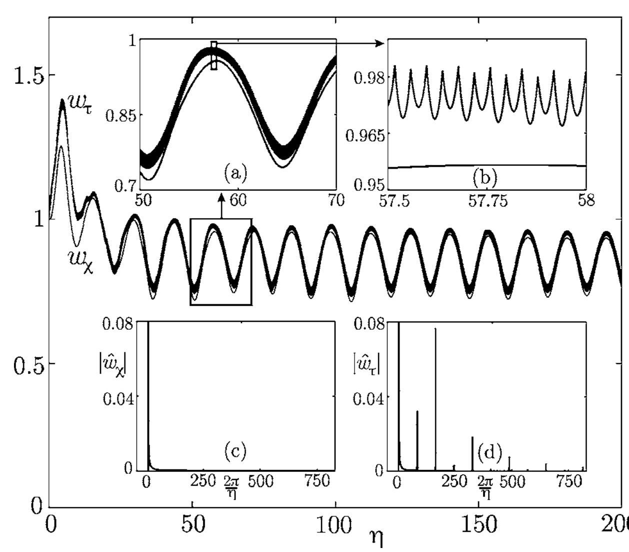

Fig. 2. Oscillations of the spatial (lower) and temporal (upper) widths of the stabilized soliton with $g=1.074g_c$ from Fig. 1. (a) and (b) Insets display details of the curves. (c) and (d) Insets show the frequency spectra of the spatial and temporal width oscillations, respectively.

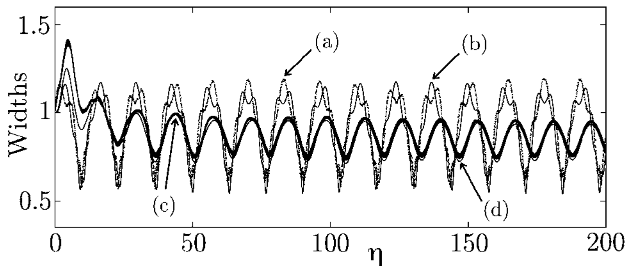

Fig. 3. Comparison between the evolutions of the spatial and temporal widths of the stabilized soliton with $g=1.074g_c$ obtained from the variational equations (5) and from the NLSE [Eq. (2)]. The modulation is of the form $d_a=1+8$, $d_b=1-8$, $\eta_a=\eta_b=0.02$. The plots correspond to (a) ω_τ (variational), (b) ω_χ (variational), (c) ω_τ (numerical), and (d) ω_χ (numerical).

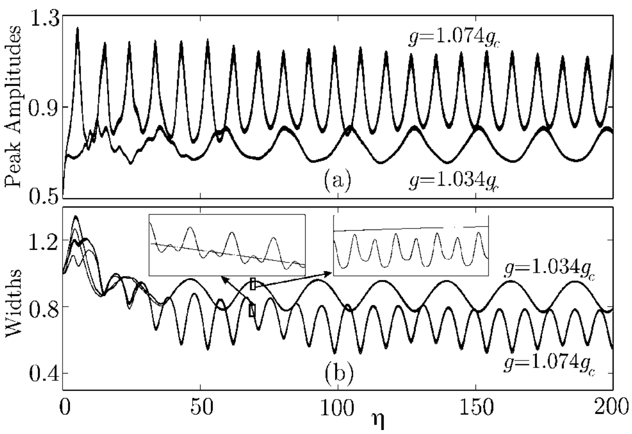

Fig. 4. Oscillations of the (a) peak amplitudes and (b) widths of the same input solitons from Fig. 1. In this case the modulation is given by $d_a=1+8$, $d_b=1-8$, $\eta_a=\eta_b=0.01$. In (b) the oscillations of the spatial and temporal widths are overlapped (insets).

ever, some changes can be appreciated when a comparison is made with the low-frequency oscillations of Fig. 1. In the first place we must stress that the amplitude of the peak oscillations is higher for both stabilized solitons; this can be understood by considering an effective stabilized soliton of higher width, compared with the layers' thickness. Thus, the effect of the nonlinearity will be a higher tendency to collapse. Concerning the frequency of the slow oscillations, we have not observed any regularity. For the soliton with $g=1.074g_c$ the frequency is multiplied approximately by $\sqrt{2}$. For the soliton with $g=1.034g_c$, the factor is approximately 3. This means that the dynamics of the slow oscillations is also dependent on the input condition.

5. ANALYSIS OF THE ROBUSTNESS

In this section we will explore the robustness of the stabilization method. To this aim, we will analyze two variations on the case of the previous section. In first place we will analyze the effect of changes in the modulation function, studying both changes in its shape by changing the type of the modulation function. On the other hand, we will investigate the stability of the solitons against the addition of noise.

A. Stabilization with Layers of Different Sizes

Let us begin with the case of two layers of different sizes. It is intuitive that the total balance of dispersion should be kept constant in order to achieve stabilization; thus if one of the layers is reduced to half its value, the value of the dispersion in this region should be doubled. Taking this into account, we show the results obtained in Fig. 5, where we have taken layers of size $2\eta_a=0.01$, $2\eta_b=0.03$. Therefore, to keep dispersion constant, we need values of $d_a=1+17$, $d_b=1-17/3$. As can be seen in the graphs, the effect in the low-frequency oscillations is quite similar to Fig. 4. Our explanation of this is that the average thickness of the layers is the same in both cases; this adds extra support to the previous qualitative explanation in terms of an effective wider soliton for thinner layers.

B. Stabilization with Harmonic Modulation

Another interesting situation corresponds to a waveguide that has been prepared to display a periodic modulation along propagation different from the step case studied previously. Although the detailed shape that could be obtained depends on the fabrication process, by one's changing the dispersion relation along the waveguide, a harmonic function is a simple choice to study the effect of different modulation functions in the stabilization process.

In Fig. 6 we show the results of numerical simulations when the dispersion is taken as $d(t)=d_0+d_1\cos(\Omega t)$ with $d_0=1$, $d_1=8$, and $\Omega=80$, which correspond to the values of Section 4. It can be appreciated that the change from a step to a sinusoidal function does not much affect the stabilization. This adds extra support to the robustness of the procedure. Compared with Fig. 1, the main differences appear in the frequency and amplitude of the low-frequency oscillations. The soliton with $g=1.034g_c$ is more affected by the change in the modulation function.

C. Effect of Noise

Finally, we will briefly study the stability of the solitons obtained against the addition of noise. This is shown in Fig. 7 where we have perturbed the initial beams with a random noise of 1% in amplitude. We have taken sinusoidal modulation with $d_0=1$, $d_1=8$, and $\Omega=157$ (which

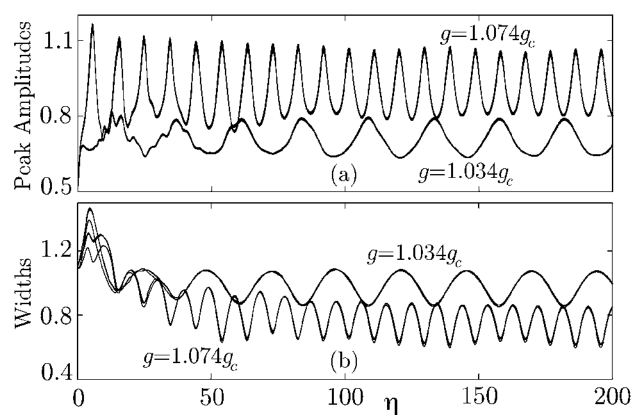

Fig. 5. Oscillations of the (a) peak amplitudes and (b) widths of the same input solitons from Fig. 1. In this case the modulation is given by $d_a=1+17$, $d_b=1-17/3$, $\eta_a=0.005$, $\eta_b=0.015$. In (b) the oscillations of the spatial and temporal widths are overlapped.

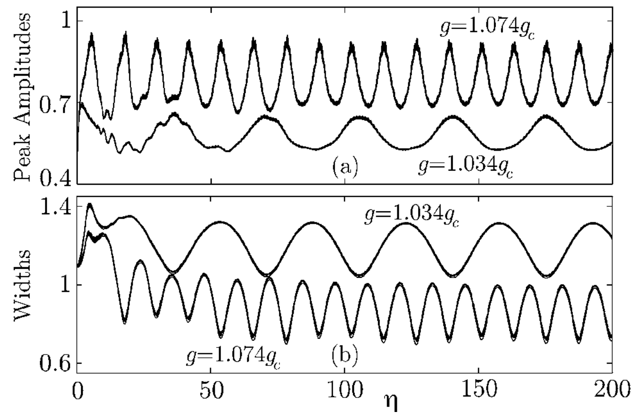

Fig. 6. Oscillations of the (a) peak amplitudes and (b) widths of the same input solitons from Fig. 1. In this case the modulation is given by a sinusoidal function with $d_0=1$, $d_1=8$, $\Omega=80$. In (b) the oscillations of the spatial and temporal widths are overlapped.

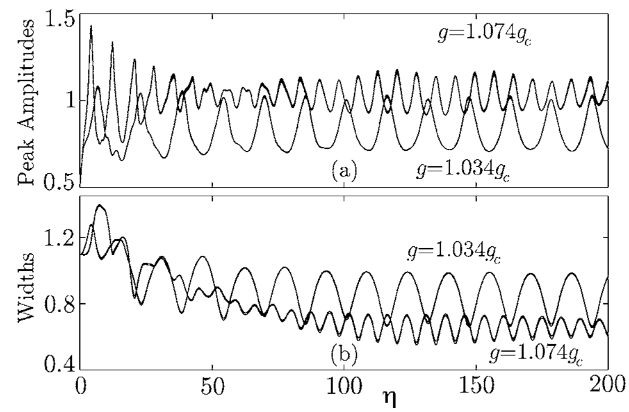

Fig. 7. Oscillations of the (a) peak amplitudes and (b) widths of the same input solitons from Fig. 1 when a random noise of 1% in amplitude is added to the initial data. In this case the modulation is sinusoidal with $d_0=1$, $d_1=8$, $\Omega=157$. In (b) the oscillations of the spatial and temporal widths are overlapped.

corresponds, in the step dispersion case, to layers of size $\eta_a=0.01$, $\eta_b=0.01$). The main effect of noise is a little disturbance in the low-frequency oscillations, which appear less regular than in the previous figures. We have checked that the stabilization is robust below a noise threshold of approximately 5% of the peak amplitude of the input signal.

6. CONCLUSIONS

In conclusion, we can say that in this paper we have described two-dimensional spatiotemporal solitons that are stabilized against collapse by means of dispersion management. We have studied their stability and properties both analytically and numerically. We have shown that the adequate choice of the modulation parameters will optimize the stabilization of the pulse. We have also checked the robustness of the method by strong changes in the modulation function and the addition of noise. Our results are of importance in the field of high-power pulse propagation in nonlinear optical materials.

A remaining open question concerns the stability of higher-dimensional beams. In fact, at the present time the mechanism of stabilization of fully three-dimensional solitons, which have been predicted to collapse in the case of modulation of nonlinearity, is not clear.[15] Thus, our results on dispersion management would be an important step forward in stabilization of light bullets in combination with modulation of nonlinearity.

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REFERENCES

1. C. Sulem and P. Sulem, *The Nonlinear Schrödinger Equation: Self-Focusing and Wave Collapse* (Springer, 2000).
2. Yu. S. Kivshar and G. P. Agrawal, *Optical Solitons: from Fibers to Photonic Crystals* (Academic, 2003).
3. V. E. Zakharov and A. B. Shabat, "Exact theory of two-dimensional self-focusing and one-dimensional self-modulation of waves in nonlinear media," Sov. Phys. JETP **34**, 62–69 (1972).
4. A. Hasegawa and F. Tappert, "Transmission of stationary nonlinear optical pulses in dispersive dielectric fibers. 1. Anomalous dispersion," Appl. Phys. Lett. **23**, 142–144 (1973).
5. S. Maneuf, A. Barthelemy, and C. Froehly, "Soliton beam propagation; space-time behaviour and spectral features," J. Opt. **17**, 139–145 (1986).
6. A. A. Kanashov and A. M. Rubenchik, "Wave mutual focusing in media with quadratic nonlinearity," Opt. Commun. **24**, 121–124 (1978).
7. B. A. Malomed, P. Drummond, H. He, A. Berntson, D. Anderson, and M. Lisak, "Spatiotemporal solitons in multidimensional optical media with a quadratic nonlinearity," Phys. Rev. E **4**, 4725–4735 (1997).
8. X. Liu, K. Beckwitt, and F. Wise, "Two-dimensional optical spatiotemporal solitons in quadratic media," Phys. Rev. E **62**, 1328–1340 (2000).
9. I. G. Koprinkov, A. Suda, P. Q. Wang, and K. Midorikawa, "Self-compression of high-intensity femtosecond optical pulses and spatiotemporal soliton generation," Phys. Rev. Lett. **84**, 3847–3850 (2000)
10. M. Segev and G. I. Stegeman, "Self-trapping of optical beams: spatial solitons," Phys. Today **51**, 42–48 (1998).
11. H. Michinel, R. de la Fuente, and J. Linares, "Temporal modulation of spatial optical solitons," Appl. Opt. **33**, 3384–3390 (1994).
12. L. Berge, V. K. Mezentsev, J. J. Rasmussen, P. L. Christiansen, and Y. B. Gaididei, "Self-guiding light in layered nonlinear media," Opt. Lett. **25**, 1037–1039 (2000).
13. I. Towers and B. A. Malomed, "Stable (2+1)-dimensional solitons in a layered medium with sign-alternating Kerr nonlinearity," J. Opt. Soc. Am. B **19**, 537–543 (2002).
14. G. D. Montesinos, V. M. Pérez-García, and H. Michinel, "Stabilized two-dimensional vector solitons," Phys. Rev. Lett. **92**, 133901 (2004).
15. H. Saito and M. Ueda, "Dynamically stabilized bright solitons in a two-dimensional Bose–Einstein condensate," Phys. Rev. Lett. **90**, 040403 (2003).
16. G. D. Montesinos, V. M. Pérez-García, and P. Torres,

"Stabilization of solitons of the multidimensional nonlinear Schrödinger equation: matter-wave breathers," Physica D **191**, 193–210 (2004).

17. L. Bergé, E. A. Kuznetsov, and J. J. Rasmussen, "Defocusing regimes of nonlinear waves in media with negative dispersion," Phys. Rev. E **53**, R1340–R1343 (1996).
18. M. Matuszewski, M. Trippenbach, and B. A. Malomed, "Two-dimensional dispersion-managed light bullets in Kerr media," Phys. Rev. E **70**, 016603 (2004).
19. V. Zharnitsky, E. Grenier, S. K. Turitsyn, C. K. R. T. Jones, and J. S. Hesthaven, "Ground states of dispersion-managed nonlinear Schrödinger equation," Phys. Rev. E **62**, 7358–7364 (2000)
20. V. Zharnitsky, E. Grenier, C. K. R. T. Jones, and S. K. Turitsyn, "Stabilizing effects of dispersion management," Physica D **152**, 794–817 (2001).
21. F. Abdullaev, B. Baizakov, and M. Salerno, "Stable two-dimensional dispersion-managed soliton," Phys. Rev. E **68**, 066605 (2003).
22. F. Smektala, C. Quemard, V. Couderc, and A. Barthelemy, "Non-linear optical properties of chalcogenide glasses measured by Z-scan," J. Non-Cryst. Solids **274**, 232–237 (2000).
23. D. Anderson and M. Lisak, "Bandwidth limits due to incoherent soliton interaction in optical-fiber communication systems," Phys. Rev. A **32**, 2270–2274 (1985).